Some Aspects of Aerial Combat

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A "dogfight" between two aircraft each represented by a point mass in a horizontal plane of constant speed with bounded turning capability is examined. A number of complexities associated with this problem are avoided by the way the capture zones for each of the players are defined. However, by so doing, the possible outcomes of mutual destruction and draw are not distinguishable from a win by each of the players. Both the methods of Qualitative Game Theory and Control Theory are used in the analysis of a particular example. Two regions of state space of interest to the first aircraft are delineated. These regions have the property that if the second aircraft is located in the first region, the first aircraft can win the dogfight; however, if the second aircraft is located in the second region, then the first aircraft may lose the dogfight.

Introduction

CONSIDER two fighter aircraft with close performance capabilities and similar weapon effectiveness zones. We will assume that each aircraft may be represented by a point-mass in a horizontal plane at constant speed with bounded turning capability. One aircraft's speed is assumed slightly higher than the other, but the advantage in speed is traded off with less turning capability. This would be the case for two modern-fighter combat with close performance such as F-4 vs MIG-21.

In an aerial duel or dogfight, each aircraft is attempting to maneuver such that the other is brought into a position or "capture zone" favorable for destruction. This immediately brings up the problem of "role ambiguity" for the pilot of each aircraft. That is, it may not be clear at any given moment whether a given participant in the combat should be playing the role of a pursuer or an evader. Indeed during the course of an engagement a participant may be required to (or should) switch roles. Ciletti and Starr (Ref. 1, p. 10) point out this difficulty and the "inadequacy" of two-person zero-sum differential game theory to handle it. The difficulty is that the analysis assumes a fixed role definition for each player and that the game theoretic capture sets do not accurately model the physical conflict.

We certainly agree with this observation. However a prudent mix of qualitative game theory, controllability theory, and choice for the capture sets does allow for at least a partial yet realistic examination of this problem. This will be the approach used here. Ciletti et al.[2] have examined this problem using reachable sets techniques whereas Olsder and Breakwell[3] have used game theory.

Each aircraft in a dogfight is more interested in positioning his opponent directly ahead of him. On the other hand, each aircraft would like to avoid being approached from behind by his opponent. Thus there is an effective fire zone in front and a danger zone behind each aircraft. There are four zones in all. If combat is said to terminate when these zones appropriately overlap we would have four possible outcomes. Designate by I and II the two adversaries, the outcomes are I destroys II, II destroys I, mutual destruction and draw.

We will greatly reduce the complexity of the analysis of aerial combat by considering only two zones—a fire zone θ_{I} and danger zone θ_{II} for the first aircraft only. This eliminates the possibility of mutual destruction and draw. If aircraft II enters θ_{I} then aircraft I wins, and if aircraft II enters θ_{II} then aircraft II wins. We note that this is not entirely realistic in view of the above comments. We are saying that the heading of aircraft II as it enters θ_{I} or θ_{II} is unimportant. Nevertheless, we will show that some of the essential features of the basic dogfight problem are contained in the results we obtain here.

Since our model does not separate the first-two defined outcomes from the possible outcomes such as "mutual destruction" and "draw" we have in effect included "mutual destruction" as a win for I and a "draw" as a win for II. A more general approach to targets and outcomes has been discussed by Baron et al.[4]

Many of the problems examined using differential game theory have used circular capture sets. For example the "two-car" problem of Isaacs,[5] "air-to-air-combat" problem of Miller[6] and "collision avoidance" problem of Vincent et al,[7] all have chosen a circular capture zone around pursuer or evader. In spite of the convenience of using circular capture zones, in a realistic aerial duel, the position of the second aircraft on a circular zone around the first does not correspond to a favorable tactical position as far as the first aircraft is concerned. Indeed the fire effectiveness zones for aircraft are best described in terms of some nonsmooth "fan-shaped" sets.

Problem Definition

We have two aircraft, I and II. For point of reference, we will consider ourselves to be on the side of I, the faster aircraft. We are thus interested in the motion of II with respect to I and will write the system dynamics with respect to a relative coordinate system.

Let x_1 and x_2 be nondimensional relative coordinates in the plane of motion. Aircraft I is located at the origin and the x_2 axis is taken to be in the direction of I's velocity vector. Let x_3 be the angular displacement of II's velocity vector with respect to the x_2 axis. In terms of such coordinates the kinematical motion of II with respect to I is given by

$$\dot{x}_1 = \sin x_3 + u x_2 \tag{1}$$

$$\dot{x}_2 = \cos x_3 - u x_1 - \gamma \tag{2}$$

$$\dot{x}_3 = u - v \tag{3}$$

where u is I's turning rate (control), v is II's turning rate (control), and γ is the ratio of I's speed divided by II's speed ($\gamma > 1$). The equations are nondimensionalized with respect to II's performance. We also have the following inequality conditions on the controls

$$|u| \leqq \delta \quad (\delta < 1) \tag{4}$$

$$|v| \leqq 1 \tag{5}$$

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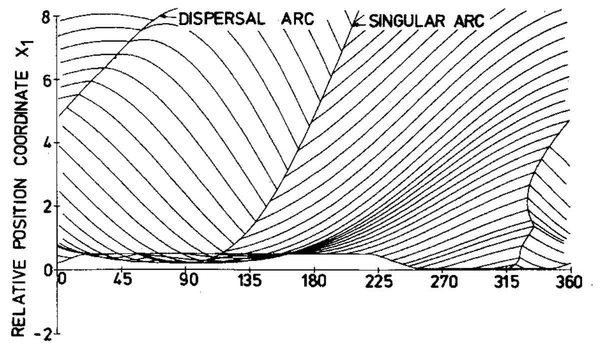

Fig. 1 Trajectories on an x_1–x_3 projection of the right game barrier.

We will take as I's fire zone a fan shaped x_3 cylindrical zone in front of I defined by

$$\theta_{\mathrm{I}} = x: \begin{cases} x_1 - bx_2 \leqq 0 \\ x_1^2 + x_2^2 - R^2 \leqq 0 \\ -x_1 - bx_2 \leqq 0 \\ -x_2 + c \leqq 0 \end{cases} \tag{6}$$

where b, c, and R are positive constants. The danger zone for I will be the fan-shaped x_3 cylindrical zone behind I defined by

$$\theta_{\mathrm{II}} = x \begin{cases} x_1 + bx_2 \leqq 0 \\ x_1^2 + x_2^2 - R^2 \leqq 0 \\ -x_1 + bx_2 \leqq 0 \\ x_2 + c \leqq 0 \end{cases} \tag{7}$$

There are a number of questions involved for the participants in our model. First, what states of the system are controllable to θ_{I} with a "win for I" outcome by I's control action no matter what II does to avoid it? To answer this question, a pursuit-evasion problem will be solved using the given target set θ_{I}: The roles to be assumed by I and II are clearly defined. I is the pursuer and II is the evader.

Similarly, a "win for II" region can also be determined by solving another pursuit-evasion problem with respect to the target set θ_{II}. In this case, I is the evader, II is the pursuer. However, since we are on I's side, we will address ourselves to a different question with respect to the region θ_{II}. The region θ_{II} is interpreted as a capture zone for player II. Insteady of finding the region of space in which player II can reach θ_{II} under optimal play by player I, we will seek a more pessimistic capture zone to insure some safety for player I. Namely we will seek the region of space in which player II can reach θ_{II} under null control by player I.

We will thus be interested in solving a game of kind with respect to θ_{I} and a problem in reachability‡ with respect to θ_{II}. In the parlance of Ref. 7, we will be seeking the red barrier for θ_{I} and the green barrier for θ_{II}. Depending on how these barriers intersect, we will have an interpretation of the possible game outcomes.

Game Barrier Trajectories Associated with the Set θ_{I}

If two game barriers§ emanating from the capture set θ_{I} intersect (perhaps at infinity), then the points enclosed by the two game barriers with the capture set may represent states on which I can guarantee to reach, no matter what II does to prevent it. If two such game barriers fail to intersect and terminate within a finite distance to the capture set, then the game barriers do not divide the state space into two parts. If this is the case, even though the game barrier does not bound a guaranteed reachable set by I, it does delineate a region in which I's task is more difficult.

From Theorem 6.1, p. 131 of Ref. 8, we obtain the following necessary conditions for a control to be a game surface control for θ_{I}

$$u = \begin{cases} +\delta & \text{if} \quad \sigma_u < 0 \\ -\delta & \text{if} \quad \sigma_u > 0 \\ 0 & \text{if} \quad \sigma_u = 0 \end{cases} \tag{8}$$

$$v = \begin{cases} +1 & \text{if} \quad \sigma_v > 0 \\ -1 & \text{if} \quad \sigma_v < 0 \\ 0 & \text{if} \quad \sigma_v = 0 \end{cases} \tag{9}$$

where

$$\sigma_u = \lambda_1 x_2 - \lambda_2 x_1 + \lambda_3 \tag{10}$$

$$\sigma_v = -\lambda_3 \tag{11}$$

$$\dot{\lambda}_1 = \lambda_2 u \tag{12}$$

$$\dot{\lambda}_2 = -\lambda_1 u \tag{13}$$

$$\dot{\lambda}_3 = \lambda_2 \sin x_3 - \lambda_1 \cos x_3 \tag{14}$$

In addition, we have the condition that at every point of a game surface trajectory

$$H = \lambda_1 \sin x_3 + \lambda_2(\cos x_3 - \gamma) + \sigma_u u + \sigma_v v = 0 \tag{15}$$

In this case, the capture set is not smooth and at "corner" points we must use the transversality condition of Peng[9] that the final value of the adjoint vector must lie in the gradient cone to the surface. A complete discussion of how the transversality conditions are used to locate terminal points and the proper terminal controls is contained in this reference.

The game barrier emanating from the right side of the capture set θ_{I} ($x_{1f} > 0$) will be termed the right game barrier, and the one starting from the left side ($x_{1f} < 0$) will be called the left game barrier. Trajectories which lie in the game barriers are determined by integrating the equations of motion (1–3) and the adjoint equations (12–14) from the capture set in the retro sense using game control pairs obtained from Eqs. (8) and (9). Integration is initiated using the control pairs and initial conditions as obtained from the transversality conditions (see also Ref. 9).

Figures 1 and 2 show, for the case where $\gamma = 2^{1/2}$, $\delta = \frac{1}{2}$, $b = 1/(3)^{1/2}$, $c = 0.05$, and $R = 1$, the projections of trajectories on the right game barrier onto the $x_1 - x_3$ and $x_2 - x_3$ plane. Trajectories on the left game barrier are similar.

Trajectories in Figs. 1 and 2 starting above and below $x_{3f} = 316.608°$ intersect. The points of intersection generate a dispersal curve whose projection is shown in the figures. It is of interest to note that the time to intersection differs depending on the choice of trajectories.

A second important curve shown on these figures is the singular arc which intersects x_{3f} at 56.785°. The nonsingular portion of a singular trajectory switches off this singular arc with non-singular control. The dispersal curve of the left game barrier is generated from $x_{3f} = 43.392°$, and its singular arc starts from $x_{3f} = 303.215°$.

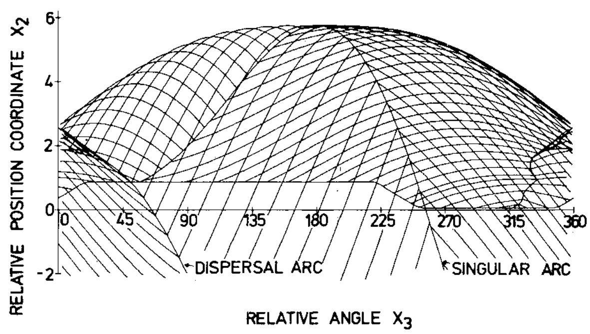

Fig. 2 Trajectories on an x_2–x_3 projection of the right game barrier.

‡ The term *reachability* is in reference to a retrograde system. The system equations are integrated backward from the capture set.

§ We use the term *game barrier* to denote surfaces which satisfy game surface necessary conditions.

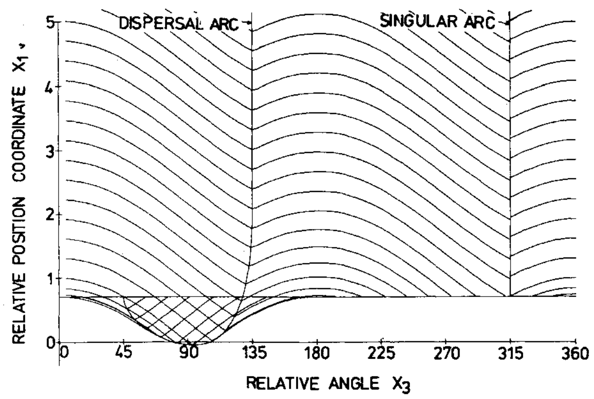

Fig. 3 Trajectories on an x_1–x_3 projection of the right reachable barrier.

Reachable Barrier Associated with Capture Set θ_{II}

The reachable barrier¶ from the capture set θ_{II} separates the states which can be reached by player II, provided that I does not perform any maneuvers (i.e., uses the control $u = 0$).

From Theorem 1 of Ref. 10 the following necessary conditions for a control to be a reachable surface control for θ_{II} are

$$v = \begin{cases} +1 & \text{if } \sigma_v < 0 \\ -1 & \text{if } \sigma_v > 0 \\ 0 & \text{if } \sigma_v = 0 \end{cases} \tag{16}$$

where $\sigma_v = -\lambda_3$

$$\begin{aligned} \dot{\lambda}_1 &= 0 \\ \dot{\lambda}_2 &= 0 \\ \dot{\lambda}_3 &= \lambda_2 \sin x_3 - \lambda_1 \cos x_3 \end{aligned} \tag{17}$$

In addition we have the condition that at every point of the reachable surfaces trajectory

$$H = \lambda_1(\sin x_3) + \lambda_2(\cos x_3 - \gamma) + \sigma_v v = 0 \tag{18}$$

The transversality conditions are again used to determine the terminal state control for all trajectories emanating from θ_{II}. It can be shown that we can rule out the possibility of trajectories starting from the smooth portion of all boundary surfaces of θ_{II} for the assigned values of all parameters (γ, δ, b, c, and R). It can also be shown that, with these assigned parameters, all extreme trajectories start from either the lower right side corner or the lower left side corner. The reachable barriers emanating from the right and left side of the capture set are called the right and left reachable barriers, respectively.

Figures 3 and 4 show the trajections onto the $x_1 - x_2$ and $x_2 - x_3$ planes of the trajectories on the right reachable barrier for the case where $\gamma = 2^{1/2}$, $\delta = \frac{1}{2}$, $b = 1$, $c = 0.05$, and $R = 1$. The left reachable barrier is similar. The dispersal curve shown in Figs. 3 and 4 is the intersection of the surface generated by trajectories which start with values of $x_{3f} > 45°$, with the surface generated by trajectories which start with $x_{3f} < 45°$. The singular portion of a singular trajectory which starts from $x_{3f} = 315°$ switches off this singular arc with nonsingular control. The dispersal curve of the left reachable barrier is generated from $x_{3f} = 315°$, and its singular arc starts from $x_{3f} = 45°$.

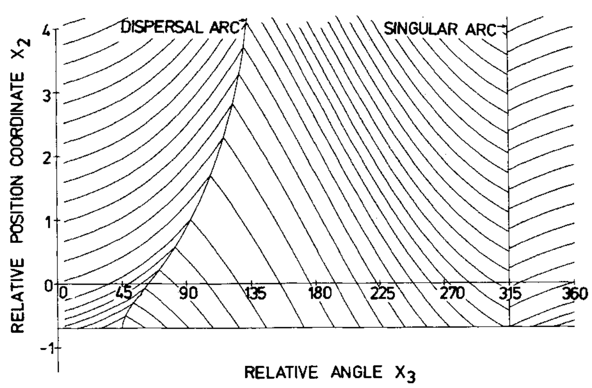

Fig. 4 Trajectories on an x_2–x_3 projection of the right reachable barrier.

¶ We will use the term *reachable barrier* to denote surfaces which satisfy reachable surface necessary conditions.

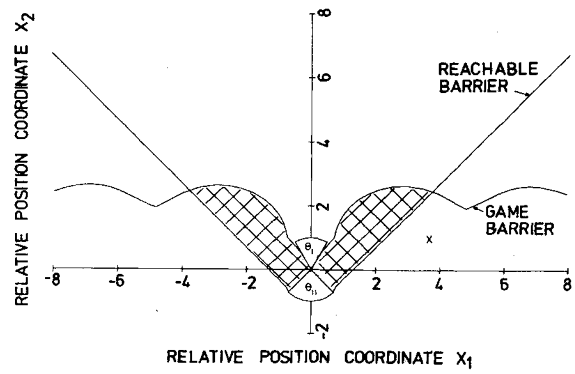

Fig. 5 Game and reachable surface cross section at $x_3 = 0°$.

Game and Reachable Barrier Cross Sections

From the family of trajectories lying on the game and reachable barriers we can now obtain barrier cross sections for a given value of x_3. Figures 5–9 are x_3 cross sections of the barriers for $x_3 = 0°$, 45°, 90°, 135°, and 180°. Any barrier cross section at $x_3 = 180° + \alpha$ can be obtained from the barrier cross section at $x_3 = 180° - \alpha$ by replacing x_1 by $-x_1$. There are two game barriers, one from each side of θ_I. Similarly there are two reachable barriers, one emanating from each side of θ_{II}. Although the capture set θ_{II} is different from the one used in Ref. 7, the reachable barrier trajectory projections (Figs. 3 and 4) obtained here agree qualitatively with the green barriers published in this reference.

From the x_3 cross-section figures, we observe that the reachable barriers from the right and left side of θ_{II} extend to infinity and "expand" in front of θ_{II}. The region enclosed by the boundary of θ_{II} and the two reachable barriers contains points which can be reached by II, provided that I does not execute any maneuvers. Note that θ_I is contained in this reachable region in most of the x_3 cross sections. It is also observed that the sides of the reachable barrriers are straight lines except in the vicinity of the capture set θ_{II}.

From Figs. 1 and 2 together with Figs. 5–9, we conclude that the boundary of the usable part of θ_I is confined to the left

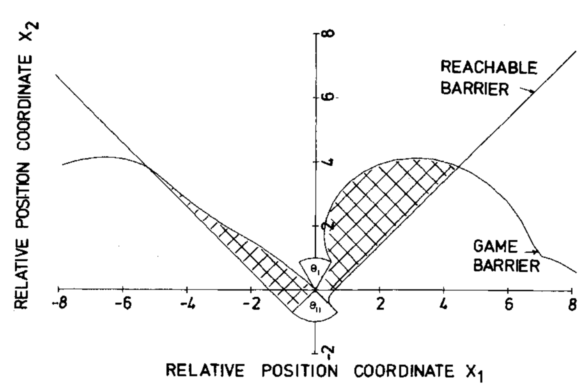

Fig. 6 Game and reachable surface cross section at $x_3 = 45°$.

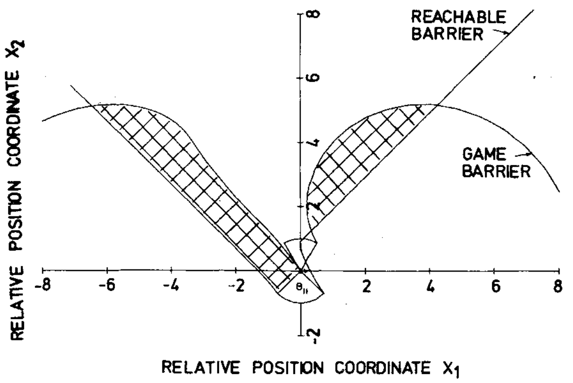

Fig. 7 Game and reachable surface cross section at $x_3 = 90°$.

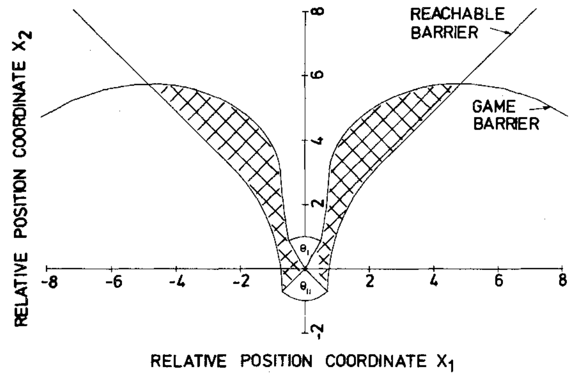

Fig. 9 Game and reachable surface cross section at $x_3 = 180°$.

and right-hand sides of θ_I. These figures also show that the usable part contains the upper surface of θ_I for all x_3 cross sections. Although the capture set is a nonsmooth fan-shaped region instead of a circular region around the origin, the game barrier obtained here resembles a portion of the barrier emanating from a circular target given by Isaacs (Ref. 5, p. 243).

From our analysis and results, we have observed that the game barriers from right and left side of θ_I neither extend to infinity nor intersect with each other. Thus, the game barriers do not divide the state space into two parts. The entire state space is always controllable to θ_I by I. To reach points such as X from θ_I of Fig. 5, II can force I into employing a strategy such that the resulting trajectory has to go around the right game barrier instead of following a direct route. In this case, the game barrier, although it does not delineate capture or escape, does mark the states to which the first aircraft must take a devious route. For the choice of parameters (γ, δ, R), this conclusion also agrees with Miller[6] who has obtained numerically a family of curves that give parameter values for which the entire space is capturable from a circular target.

The sides of the game barriers, unlike those of the reachable barriers, are always curved because I is active on such a barrier. The "corner" on the game or reachable barrier is a point on the dispersal curve of the respective barrier. Player II's control is discontinuous across the corner; that is, II turns hard right on one side of the corner while he turns hard left on the other side.

One of the major differences between the game and reachable barriers is that the right and left game barrier terminate within a finite distance from the origin. But before a game barrier terminates in state space, it intersects with a reachable barrier. Thus, the state space is separated into two types of zones. One zone is cross-hatched as shown in Figs. 5–9 and represents a zone of caution to player I. The other zone is the rest of the state space not including the capture sets θ_I and θ_{II} and should be capturable by I, if he plays optimally; that is, if he uses proper controls even he has to swing all the way around the game barrier. In the process of reaching such states, there is no danger of II reaching θ_{II} first. On the other hand, if an attempt is made by I to capture II when he is in the zone of caution, there is a distinct possibility that II may reach θ_{II} first. Thus I should guard against any maneuvers that would allow II to enter the zone of caution.

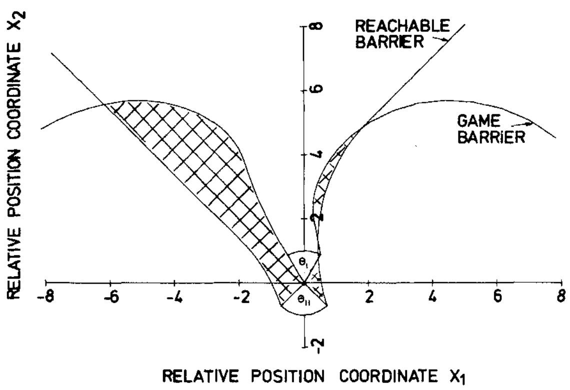

Fig. 8 Game and reachable surface cross section at $x_3 = 135°$.

Discussion of Results

Even with the simple dynamics used here, it is clear that the results are not complete. The possibility of mutual destruction and draw have not been distinguished from the win possibilities. However within the context of our analysis, we could lump mutual destruction, draw, and a win by II all into the zone of caution. What would be required is to make the target set x_3 dependent.

There are practical implications of these, or similar results which may be noted. Consider an aircraft equipped with both forward and rearward "looking" radar. The reason for the rearward looking radar is so that a pilot can be kept informed as to when another aircraft is in or near his danger zone.

Since the zone of caution presented here represents positions from which aircraft II can possibly get into I's danger zone, the pilot of I can have advance warning of a threat by II. What would be required is for I to have radar of sufficient sophistication to determine the relative heading and speed of II. An onboard computer could be used to operate a display similar to Figs. 5–9. The barriers in the display would continually change as each aircraft maneuvers. The pilot of I would observe the location of II on the display. If II is outside the zone of caution then I is assured, by appropriate maneuvering, of capturing II. If II is inside the zone of caution then I must perform an avoidance maneuver. (Maneuvers for each of the zones have not been determined; however appropriate maneuvers can be synthesized by extensions of the methods used here (see Ref. 7). By using a zone of caution display, it appears that the need for rearward looking radar can be de-emphasized.

The fighter pilot who has played the real dogfight game has many intuitive notions in regard to dangerous situations with respect to an adversary of known performance capability. In our case, I is faster yet less maneuverable than II. The 0° and 180° zone of caution cross sections seem to agree with pilot descriptions of unfavorable positioning of the adversary. However, experience is not very reliable for predicting the x_3 dependence of this zone. It goes without saying that the analytical approach can avoid wide margins of error in such predictions and allow for a quick evaluation of the situation when the adversary may be one of a number of aircraft of given performance capabilities.

The applicability of the results presented here to an actual aerial combat situation is obviously restricted by the assumptions of planar motion and constant speed. The constant speed assumption would appear to be applicable for battles of only a few seconds or so. If the participants choose to do battle in the vertical plane as well (in which large speed changes can quickly be made) then the results are definitely open to question.

Relaxing the assumptions of constant speed and horizontal motion will increase the number of state variables required for the analysis. It remains to be shown whether the type of analysis used here applied to problems of higher dimension will be as manageable.

Finally it should be noted that the results presented here seem to be in agreement with the reachable sets results of Ref. 2. A direct comparison with the results of Ref. 3 cannot be made because of the distinctively different capture sets used; however portions of the barriers obtained here are similar to the results of this reference.

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Announcement: 1974 Author and Subject Indexes

The indexes of the four AIAA archive journals (*AIAA Journal, Journal of Spacecraft and Rockets, Journal of Aircraft,* and *Journal of Hydronautics*) will be combined and mailed separately early in 1975. In addition, papers appearing in volumes of the *Progress in Astronautics and Aeronautics* book series published in 1974, as well as technical papers published in the 1974 issues of *Astronautics & Aeronautics,* also will be included. All subscribers to the four *Journals* are entitled to one copy of the index for each subscription which they had in 1974. All others may obtain it for $10 per copy from the Circulation Department, AIAA, Room 730, 1290 Avenue of the Americas, New York, New York 10019. **Remittance must accompany the order.**

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